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Inverse quadratic interpolation

Introduction

- In this topic, we will
  - Introduce the idea of using interpolating quadratics
  - Discuss approximating the inverse of a function
  - Describe how to use the approximation of the inverse to approximate the root
  - Describe the rate of convergence

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Summary

- Following this topic, you now
  - Understand inverse quadratic interpolation
    - It uses quadratic interpolating polynomials to approximate the inverse
  - Understand that this approximation of the inverse is evaluated at zero
    - This results in simply the constant coefficient
  - Know that this method is $O(h^\mu)$,
    which converges almost as quickly as Newton's method

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Introduction

- Suppose f is a function, so $y = f(x)$
  - In this case, if x is a root, then $f(x) = 0$
  - Suppose f is locally invertible around the root
  - In this case, $f^{-1}(y) = x$
    therefore $f^{-1}(0)$ must be a root
- For example,

  $\cos^{-1}(0) = \pi/2$ and therefore $\cos(\pi/2) = 0$

  $e^0 = 1$, and therefore $\ln(1) = 0$

$$y = 4x^3 + 6x + 1$$

$$x = \frac{\sqrt[3]{y-1+\sqrt{y^2-2y+9}}}{2} - \frac{1}{\sqrt[3]{y-1+\sqrt{y^2-2y+9}}}$$

$$\frac{\sqrt[3]{2}}{2} - \frac{1}{\sqrt[3]{2}}$$

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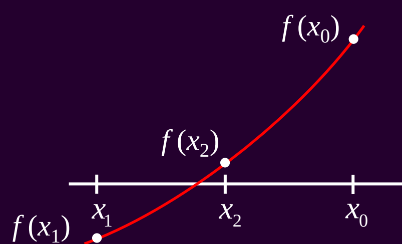
Interpolating quadratics

- A function is locally invertible if it is locally one-to-one
  - How do we approximate the inverse?
  - Find the polynomial interpolating the points

    $$(f(x_0), x_0), (f(x_1), x_1), (f(x_2), x_2)$$
  - The *f*-values must be unique, otherwise it is not invertible



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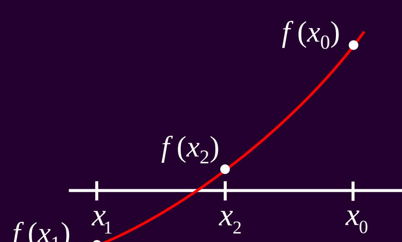
Interpolating quadratics

- Once we find the interpolating quadratic polynomial approximating the inverse
  - To find the root, we evaluate this interpolating polynomial at 0
  - That is, all we need is the constant coefficient



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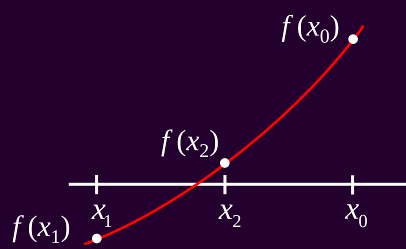
Interpolating quadratics

- The constant coefficient is

$$x_{k+1} \leftarrow \frac{\begin{array}{l}\left(f(x_k)-f(x_{k-1})\right) f(x_k) f(x_{k-1}) x_{k-2} \\ \quad +\left(f(x_{k-1})-f(x_{k-2})\right) f(x_{k-1}) f(x_{k-2}) x_k \\ \quad +\left(f(x_{k-2})-f(x_k)\right) f(x_{k-2}) f(x_k) x_{k-1}\end{array}}{\left(f(x_k)-f(x_{k-1})\right)\left(f(x_{k-1})-f(x_{k-2})\right)\left(f(x_{k-2})-f(x_k)\right)}$$



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Interpolating quadratics

- Of course, this technique fails if any two of the *f* values are the same or very similar
  - To ensure the *f* values are likely to be different, this technique works well if the root is bracketed
  - In this case, at least two of the differences are guaranteed to be relatively large
  - This is used in the next algorithm: the Brent-Dekker method

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Rate of convergence

- Like Muller's method, if h is the error, then the rate of convergence is $O(h^\mu)$ where μ is the real root of

$$x^3 = x^2 + x + 1,$$

so $\mu \approx 1.8393$

  - This is, again, significantly better than the secant method
  - If we are bracketing the root, this even better than either of the bisection or bracketed secant methods

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Inverse quadratic interpolation

References

[1] https://en.wikipedia.org/wiki/Inverse_quadratic_interpolation

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Inverse quadratic interpolation

Acknowledgments

None so far.

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Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using `Consolas`.
Mathematical equations are prepared in MathType by Design Science, Inc.
Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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